Uncertainty analysis, a necessity?

Uncertainty in measurements and the propagation of uncertainty can twist the fundamentals, when interpreted out of context and with minimal understanding

Uncertainty analyses in engineering profession is often seen as an option, more often discussed in research studies and data analyses, yet, is it actually the case? It is merely included in the engineering professional assignments, intertwined in basic computations. For example, the concept of characteristics strength of concrete (and the mean strength is not the design parameter).

We may recall, an announcement from OPERA physicists startled the scientific community, when they proclaimed to have found neutrino particles traveling at speeds higher than the speed of light. This announcement was made on the 24th of September, 2011, where the physicists themselves were in disbelief as this could be hardly be true. According to Einstein's special theory of relativity, no particle can travel at a speed of light (approximately 299,792,458 m/s). Was the discovery to be proven correct, modern theories on physics may had to be re-written, just like Newton's theory of gravity was when Einstein proposed the space-time model of the universe. However, it was later observed to have been erroneous, and the error was attributed to an obvious mistake in calculating the travel time of neutrino particles. The findings of the experiment was brought under so much scrutiny, mainly because of the fact that it challenged a fundamental scientific belief. Later on it was declared that the uncertainty in the measurements and propagation of uncertainty contributed to an error, when rectified, the challenge was faltered. It is understandable that even at the top end researches, the uncertainty analysis is critical and vulnerable, while a mishap could lead to unprecedented errors, defaulting fundamentals.

Engineering is a field of profession that relies fundamentally on measurements and applications are designed based on measurements. Engineering metrology is a subject often taught in undergraduate curriculum for all engineering disciplines, which encompasses the science of measurements. All measurements inherently constitute error, which is defined in books as the difference between the actual value and the measured value. However, in my view, if you know the actual value, we may never have to take measurements, and in case we have the option of getting the actual value, we may never resort to taking measurements either. What is an error and how is it defined? I prefer to use the term, **UNCERTAINTY** instead of error in engineering measurements. When we take measurements of a variable, we will have to make several measurements (repeated measurements, or replicates) to determine the mean and standard deviation. Suppose all systematic errors in the measurements are wrung out, the mean of the measurements could then be postulated as the actual value of the measurement, while the standard deviation would dictate the uncertainty of the measurement obtained.

Figure 01: Uncertainty: normally distributed

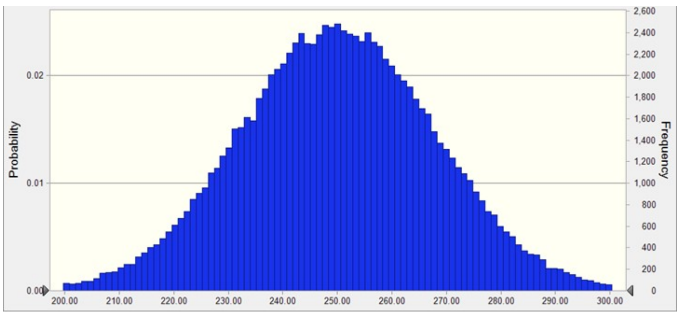

Figure 01 shows the histogram of several repeated measurements (100,000 in this example) taken of a variable. The right-side axis shows the frequency of a value of a bin considered where the left side shows the corresponding probability. It could be observed that the probability of the measurement being the mean of the observations (approximately 250) has the highest probability, yet, it is merely a 2% chance (out of 100,000 measurements taken). A value of mean therefore rarely defines or represents the actual value, where as 98% of the chances are that the measured value is not the mean value.

This is where the statistical confidence plays an important role, and delineates how confident are we in reporting a measured value. Suppose I report only the mean value, my confidence would be approximately 2%. With this scenario, I would never be able to find a single value that I can report the measurement with higher confidence than the mean. How else could I increase the confidence? It is most certain that I will have to report the measurements as a range of values to improve my confidence in reporting. Before we could look into ascertaining the range of values with indicative confidence, we need to define the probability distribution function (PDF). Based on the histogram, we can define the probability distribution function (PDF) for the measurement we obtained, as shown by the red -curved-line in Figure 02.

The characteristic of the PDF is as such that the area below the PDF curve would be 1. The probability of a single value being the measured parameter would therefore be the value it indicates on the PDF curve (eg. 250 has a probability value of approximately 2.6%). The value indicated by the straight line on the left (5% line) corresponds to a reading of 220.51. This implies that the probability of the measured value being a value less

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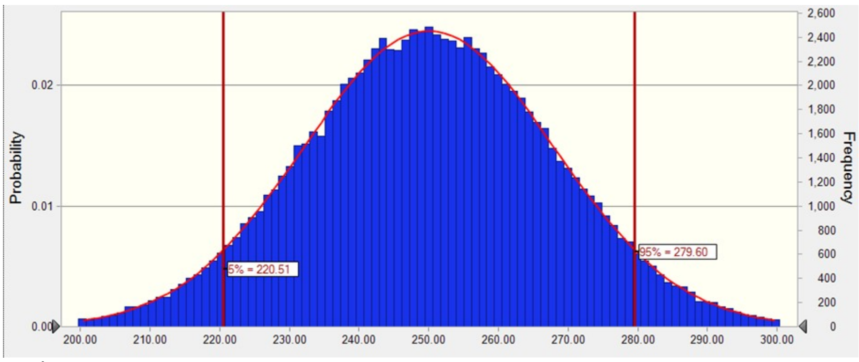
than 220.51 is 5%. Similarly, the straight line on the right (95% line) corresponds to a reading of 279.60 which implies that the probability of the measured value being higher than 279.60 is 5%. It is therefore reasonable to say that the probability of the measured value being within 220.51 – 279.60 is 90%. In other words, the measurer is 90% confident that the measured value would be somewhere between 220.51 – 279.60. Suppose the measurer wants to increase his confidence in reporting, he would have to increase the range, while decreasing the range would decrease confidence level.

In the example discussed above, the PDF was assumed to be a normal distribution, which is characteristically symmetric about the mean value. Generally, assuming the uncertainty arising from random error, a normal distribution is considered to be a good estimator for uncertainty. However, not all distributions could be approximated to a normal distribution (could be a log-normal, uniform, triangular, trapezoidal etc). Of the other distributions available, another important distribution is the log-normal distribution, which in contrast to normal distribution, is characteristically asymmetric about the mean. This uncertainty distribution in such cases would not be equal on either side of the mean. Figure 03 shows an example where similar number of measurements were taken of another engineering parameter, indicated

Figure 02: Uncertainty and confidence interval

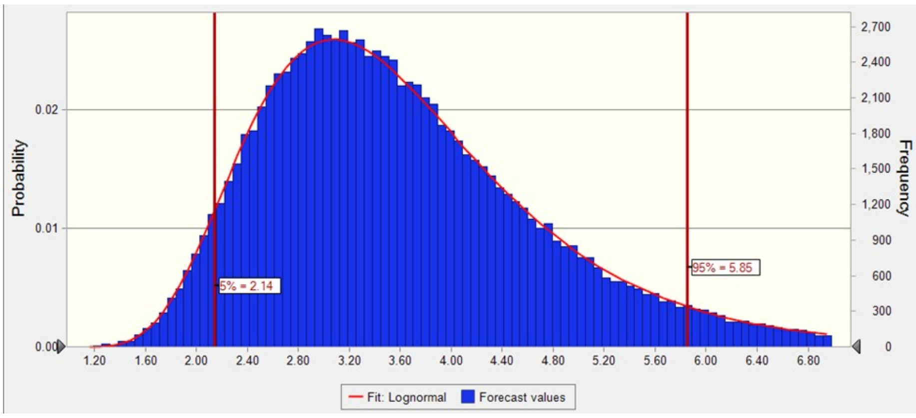


by a mean value of approximately 3.0. In contrast to Figure 02, the PDF curve in Figure 03 could be observed to be skewed to the left (shorter tail on the left and longer tail on the right). In such occasions, the uncertainty of the engineering parameter in consideration needs to be analysed with caution, should a normal distribution be approximated to represent the uncertainty.

REPORTING A MEASURED/ANALYSED VALUE

In engineering, we often will have to report our findings in terms of a measured value or a value obtained through computational analysis. As discussed above, when we report a value, we will have to give the mean and the standard deviation or a confidence interval. In this case, it is referred to as a two-tailed analysis of uncertainty, as the uncertainty may sway either side and that both boundaries will need to be reported. In addition, more often, it is assumed that the distribution of uncertainty is symmetric about the mean (normal distribution) and hence the notation often is in the form of **mean ± interval**. In real time analyses, the distribution is not always symmetric, and hence this notation may have to be adopted to include two different values for (+) and (-) about the mean.

Figure 03: Uncertainty and confidence interval (log-normal distribution)



Often in our engineering analysis, the numerical values are subsequently used in decision making. The form in which the numerical values are reported with uncertainty (two-tailed) is not applicable to the decision making exercises. I will consider two popular civil engineering related examples to elaborate the case.

Case I: Characteristics strength of concrete

We are well aware that an exercise of concrete design would result in specifying the concrete strength required for a structural element analysed in the design. If we assume that the column element needed concrete of a minimum strength of 30 MPa, the field engineer would have to cast concrete that would yield a strength of at least 30 MPa.

It is apprehensible, that the strength of concrete depends on several factors, of which the constituent fraction is just one. Designing the constituent fractions and the casting protocols would be expected to meet the 30 MPa bench mark assuming the influence of other factors are curbed.

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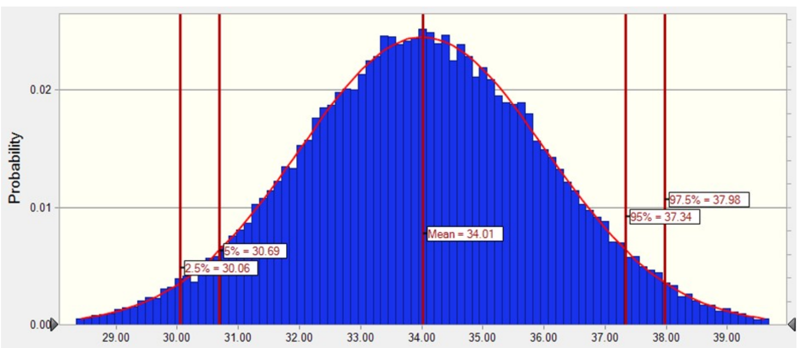
How certain are we that the resulting concrete would meet the minimum requirement of 30 MPa? Here comes the uncertainty analysis.

Sample cubes would be cast with the mix design and casting protocols, and the cubes would be tested for compressive strength after 28 days of curing, as per the standard testing methods. The compressive strength of the sample cubes would then be measured and the mean and standard deviation would be determined, assuming all samples to be replicative of each other (belonging to the same mix design and casting protocol). A normal distribution would then be fit taking into account the mean and the standard deviation computed above. Figure 04 shows the PDF fitted for the compressive strength of 100 concrete samples cast with the same mix design and casting protocols. The statement to report the strength of the concrete mix would then be 34.01 ± 3.97 MPa (considering the 2.5% and 97.5% lines, as the area enclosed would be 95%). This arises from the two-tailed analysis of the uncertainty observed in the compressive strength of the concrete mix in consideration.

The question in the exercise is more than just reporting the strength of the concrete. We need to make sure that the concrete mix designed would have a minimum compressive strength of 30 MPa to meet the design requirements for the column element. Statistically speaking, we will never be able to design a concrete mix that would always (100%) be above 30 MPa in strength, yet, we can certainly reduce the probability of having a mix less than 30 MPa. In our quest to design a concrete mix that would have a minimum strength of 30 MPa, we expect a 95% confidence with which we can design the mix. That is, out of 100 sample cubes we cast using the mix design and protocol, we would be satisfied if 95 cubes performed better than 30 MPa in compressive strength. Now going back to Figure 04, we will have to look for the line that would define the area of 5% from the left (which is indicated by a value of 30.69 in Figure 04). Therefore, it is evident that the concrete mix considered in this exercise has 95% confidence in attaining the minimum requirement of 30 MPa (in fact, even our confidence could be more than 97.5% as the 2.5% line is still above 30 MPa). This analysis is called one-tailed, as we did not consider the other end of the uncertainty spectrum. This value (the lower boundary of 95% confidence) of compressive strength is termed as the characteristics strength of concrete. Just for elaboration, if we look at Figure 05, the mean of the strength of concrete is similar to that observed in Figure 04. However, the spread of the plot is wider in Figure 05, significantly changing the uncertainty profile of the strength. As a result, the 5% confidence line on the left of the curve indicates a value of 28.90 MPa, which is less than the minimum requirement of the design. Hence, this batch of concrete mix would have to be rejected for application.

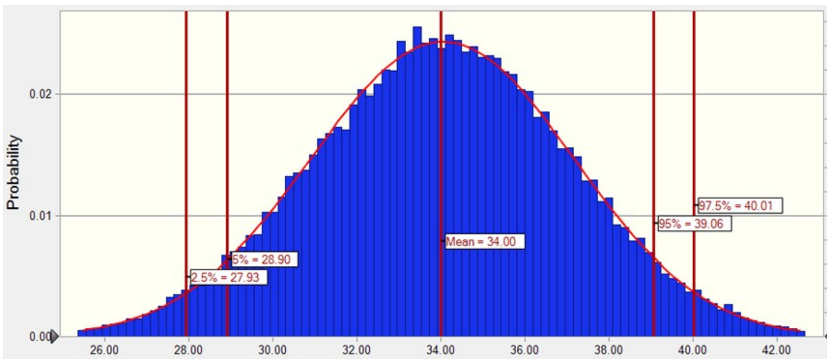
Figure 04: Strength of concrete samples



Case II: Risk analysis of pollution

Environmental engineers often have to analyse the risk of a pollution or a contamination event. This example refers to the determination of risk of a contaminant based on the concentration in water sample, with 95% confidence. We may have to take several samples of water from the suspected water resource, in order to define the uncertainty in the concentration measurement of the contaminant. Let us assume that the concentration of the contaminant is similar in values shown in Figure 04 (mean of 34 ppm). If we are to report the concentration of pollutant, we would resort to the form of two-tailed analysis of uncertainty and state the concentration is 34.01 ± 3.97 ppm. However, the exercise here is to make a decision if the contamination is of a risk. In this exercise, we will have to be 95% confident to report if the contamination is of a risk. It is always

Figure 05: Contaminant concentration



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the worse case scenario that needs to be taken into account in such decision making processes. In this example, we will need to be sure that at least 95% of the samples obtained were less in concentration than the risk level denoted by authorities. Therefore, in contrast to the one-tailed analysis considered in the concrete example where the left side of the range was considered, we will still have to look at a one-tailed analysis, yet on the right side of the spectrum this time. Suppose, a concentration of the pollutant more than 38 ppm is defined dangerous by the authorities, considering Figure 04, we could observe that 95% of the area was less than 37.34 ppm. This indicates that we can be 95% confident that the pollutant is not of a risk. However, looking at Figure 05, we could not be of 95% confidence as the line that represents 95% is 39.06 ppm, which is above the threshold set by the authorities. It is therefore understandable that the uncertainty analysis needs to be prudently handled and interpreted based on the cases, specific to the conditions and objectives of the exercises. This brings us to the next form of uncertainty analyses in engineering discipline. Uncertainty of measurements and numerical values propagate as the values traverses through functional computations. This phenomenon is called propagation of uncertainty and it is crucial in making decisions.

PROPAGATION OF UNCERTAINTY

Measurements are taken for subsequent analyses that often involve computational functions. For example, we may be interested in quantifying the moisture content of a soil sample, where the measurements would include, the weight of wet soil and dry soil with pan and the weight of pan. Each measurement (three in total) would have an uncertainty. Suppose we took only one sample from the field to quantify the moisture content, and we take one reading for each value, what is the probability that each value we measured was correct? This necessitates taking multiple measurements on multiple replicative samples (in this analysis, the wet soil once dried, cannot be used again, requiring multiple samples). From each sample, we could define the mean and standard deviation, and hence on assuming a normal distribution, could define the uncertainty in each measurement (three weights). It is apparent at this point, that the ultimate goal of the exercise was to compute the soil moisture content, which depends on the three weight measurements taken with the uncertainty defined. The function to compute soil moisture content would be,

$$MC = \frac{(M_{wet\ soil} - M_{pan}) - (M_{dry\ soil} - M_{pan})}{(M_{wet\ soil} - M_{pan})}$$

Let us assume the following values for the variables measured,

Weight of wet soil + pan = 256.25 ± 16.52
Weight of dry soil + pan = 206.46 ± 18.92
Weight of pan = 124.26 ± 35.23

Had we not considered the uncertainty in the measurements, and computed moisture content based only on the average (mean) values for each measurements, **the computed value would have been 0.377.** The question then arises is, how confident could we report this as the soil moisture content, knowing that each measurement had significant uncertainty. With a similar observation and assuming that the probability of a measurement being the mean value is approximately 2% for each measurement taken in this example, we could compute the probability of the soil moisture content computed based on means alone may be much less than 1%. The value we would then report would not even have 1% confidence. The uncertainty of each measurement propagates through the functional computation together with the mean value , resulting in an uncertainty range for the soil moisture content. Propagation of uncertainty through computations are analysed through various methods, including the popular calculus method. Discussion on the methods employed to analyse propagation of uncertainties is beyond the scope of this article, yet, I will take one of the method to discuss the factors that affect the propagation of uncertainty.

Consider a function of multiple variables, Z = f(A, B, C, …). The uncertainty in Z will have multiple components arising from each variables (changing each variable while others are kept constant). When uncertainties in variables all variables are independent variables, the total error in is given by,

$$(\alpha_Z)^2 = \left(\frac{\partial Z}{\partial A}\right)^2 (\alpha_A)^2 + \left(\frac{\partial Z}{\partial B}\right)^2 (\alpha_B)^2 + \left(\frac{\partial Z}{\partial C}\right)^2 (\alpha_C)^2 + \cdots$$

α_Z, α_A, α_B, α_C,… are uncertainties in Z, A, B, C…. respectively. It is therefore understandable that the uncertainty of a value in Z would not only depend on the uncertainty of the values of each variable (α_A, α_B, α_C …), but also on the partial derivatives of the function Z with respect to each variables. Mathematically speaking, we are aware that the partial derivatives would also include the constants in the functions attached to variables and their transforms, depending on the variable form. It is therefore clear that the uncertainty in Z would have significant impact from the functional arbitrary constants together with the form of variables and their composite functional forms.

This takes us to the last section of this article, the impact of uncertainty propagation

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in research and why experimental design is critical in research. I will discuss through my personal experience. In a research study on pervious concrete, we designed the experiments with the objective of analysing the porosity of samples with different mix designs. One of the method employed in the study to measure porosity of concrete samples was the water displacement method (the volume of water displaced when the sample is fully immersed equals the volume of solids in the concrete sample). As such, the difference in the water level had to be measured.

Taking linear measurements with a ruler was the option considered, while the ruler used for the measurement had a least count of 1 mm. Theoretically the least count of the measurements could be then approximated to 0.5 mm in water levels measured. The dimensions of the water bath was designed to be 300 mm in length and width, amounting to a cross-sectional area (plan view) of 0.09 m^2. The concrete samples were of dimensions 150 mm cube, with a volume of 0.003375 m^3. The least count of porosity measurement would correspond to therefore, 0.5 mm rise in the water level, accounting to approximately 1.3% porosity change.

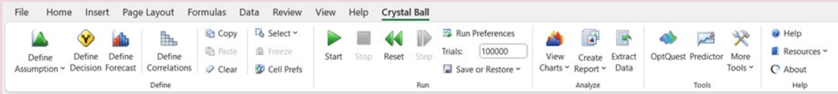
Least count of measurement also means that any measurements in between would be approximated to 0.5 mm (up or down, with eye estimation). In that case, the least count could also be considered as a definite uncertainty, on top of other random uncertainties embedded in the measurements taken. This could mean that the uncertainty in porosity measurement would be at least 1.3%, which translates to 2.6% as a range around the mean. The porosity of pervious concrete could change between 15 — 35% depending on several mix design and mixing protocols,

TOOLS & TRICKS

Propagation of Uncertainty

- Crystal Ball by Oracle

Figure a



The software developed by ORACLE, known as Crystal ball, facilitate the analysis of uncertainty propagation through functional computations through number of trials. This is an add-on to MS Excel, making it available for analysis on the excel worksheets that could be used for computational exercises. Figure a shows the add-on Crystal Ball in MS-Excel, and the tools available for analyses. The software can do several analyses on the data and uncertainty, but this article will discuss the basics of the uncertainty propagation.

STEP 01: Variables

Identify the independent variables and dependent (forecast) variables. The forecast variable must be predicted from independent variables by a computational function. Eg., consider soil moisture content. Figure b, variables in red are independent variables; variable moisture content (dependent variable) was computed from The values in the first row show the mean values measured and the values in the second row show the standard deviation.

Figure b

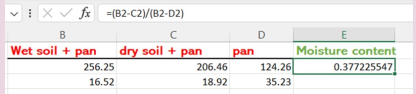
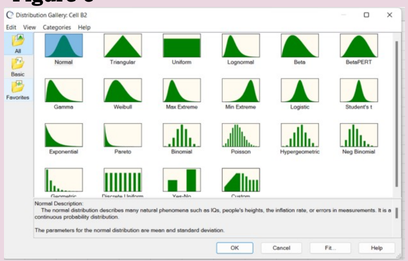


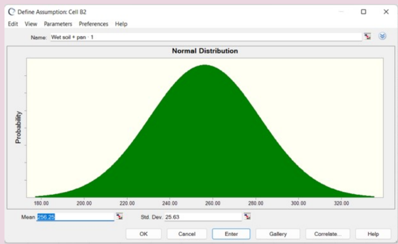
Figure c



STEP 02: Define distributions

Next step would be to define the uncertainty for each variable. Select the cell that needs to be assigned uncertainty, and then click on 'define assumptions' from Crystal Ball tool box. Figure c dialogue box will open. To define an uncertainty arising from random error, a normal distribution would suffice, while other forms of PDFs would be required when the distribution of uncertainty for the measurement is known.

Figure d



STEP 03: Define assumptions

After selecting the normal distribution for the uncertainty in the previous step, dialogue box shown in Figure d opens up. By default, the tool recognizes the selected cell for mean and it computes 10% of the mean value as standard deviation, which can be altered. By clicking on the small icon next to the text box for mean and standard deviation, we may assign values from an excel sheet, or with convenience, we could enter numerical value directly into the text boxes. The worksheet cell that is being assigned uncertainty is shown in the top bar of the dialogue box, with the name of the variable in the text box next to name. Once you have assigned the uncertainty, the corresponding cell in worksheet would turn green. Likewise, we will need to assign the uncertainties for the remaining independent measurements.

Figure e

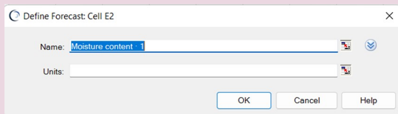
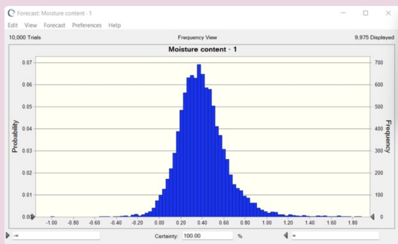


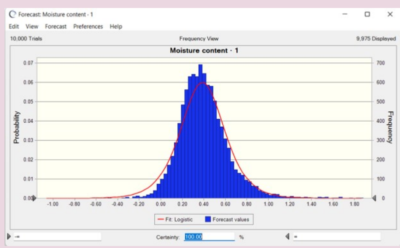
Figure f



STEP 04: Define forecast

Define prediction variable. By selecting the dependent variable (eg., the moisture content mean) we would be led to the dialogue box shown in Figure e. Unless we would want to change any of the items shown, we may accept the entries to follow.

Figure g



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and that 2.6% uncertainty would be significant (the change in porosity with mix designs may be less comparable or less than 2.6%). Models developed to predict porosity from mix designs and mixing protocols would therefore have a very high uncertainty, resulting in rejection of the model.

A solution for this would be to consider the measurement methods and tools prior, considering the uncertainty analysis. Based on the function using the calculus method, we may compute the accuracy required in measurements from the accuracy envisaged in the modeling parameters. For example, for an accuracy of approximately 0.1% porosity, we may need a water tank of dimensions of 200 mm by 200 mm with a least count measurement of 0.1 mm in the vertical scale (measuring the displacement of water).

In conclusion, it is important to have a fundamental understanding of the data that the study would include and that planning the experiments with the accuracy of model predictions expected in designing experiments are critical in successfully carrying out a research study. This would necessitate analysis of uncertainty including uncertainty propagation. In addition, analysis of uncertainty assist in making confidence level and confidence bounds for predictions, that would in turn help define the application potential or scope of application in subsequent exercises. Uncertainty analysis is therefore a necessity and not a choice.

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STEP 05: Assign trial properties

In the tool bar shown in Figure a, define the preferred number of trials in the section, 'Run'. Eg. 10,000. The software, would now pick 10,000 values for each independent variable, according to the PDF defined in STEP 03. For example, if 10,000 trials are sought, and the PDF defined has a probability of approximately 2% for the mean value, approximately 200 values would be assumed close to the mean. Simultaneously, the probability of the value at one standard deviation away from the mean value would have less than 0.2 % and that in 10,000 trials, only 20 values would be assumed close to the that value. These values are randomly assigned between the independent variables (not coordinated) and the software computes the soil moisture content in each trial. It then plots a histogram for the forecast variable as shown in Figure f.

STEP 06: PDF fitting

By clicking on the forecast tab on the dialogue box shown in Figure f, and select 'Fit probability distribution', which will automatically select the best representing probability distribution function for the histogram obtained in Figure 09. Figure g shows the logistic PDF fitted to the above obtained data.

STEP 07: Confidence limits

By double clicking the red-curved line, the dialogue box shown in Figure h could be opened. In this box we could choose the aspects of the distribution we would want to see displayed. For example, checking the check box, 'Percentile' and then selecting custom and entering 2.5 and 97.5 in the text box will give the range of values between which 95% of the area of the PDF is encompassed. From Figure i, it could be observed that the area below -0.11 and the area above 0.88 are each 2.5% of the PDF, leaving 95% of the area between -0.11 and 0.88. Therefore, it could be stated with 95% confidence that the soil moisture content of the sample is between -0.11 and 0.88. Assuming symmetric property of the PDF, this could be approximately given as 0.385 ± 0.495 (mean with 95% confidence interval). However, this notation may have two different values, should the PDF is asymmetric in nature.

STEP 08: Effect of trials

It is important to talk about the number of trials, and the impact it may have on the accuracy of the computation of soil moisture content and the uncertainty related to it. Consider the same example with 100,000 trials and 1000 trials. Figures j and k show the corresponding plots. The plot gets smoother, however, as the number of trials are increased, the running time for the trials would be significantly increased.

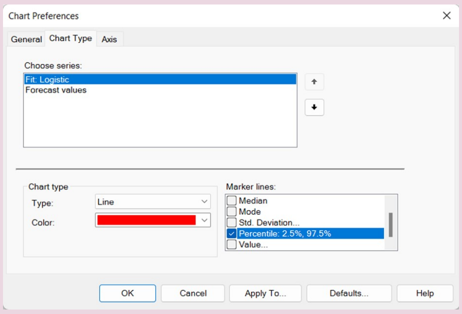


Figure i

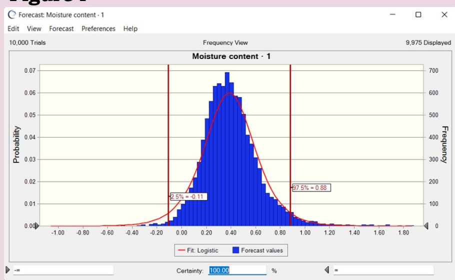


Figure j

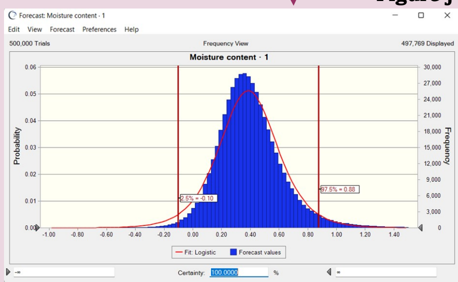


Figure k

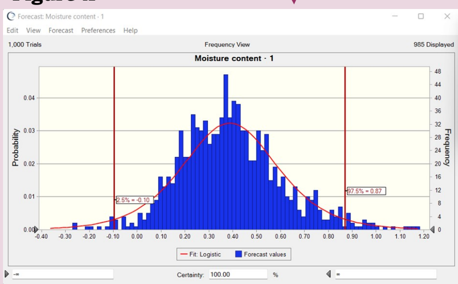
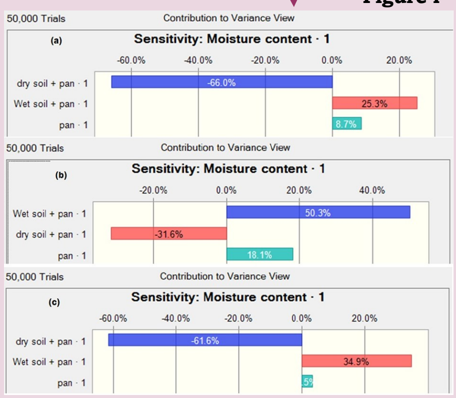


Figure l



STEP 09: Sensitivity analysis

Figure l shows the sensitivity analysis of the model of soil moisture content. The sensitivity of a variable significantly depends on both uncertainty of a variable and the function. (a) has uncertainty as assigned, (b) has uncertainty of wet soils weight reduced to half and (c) has uncertainty of 10% of the variable value.

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